Simmetrie e Anomalie

Dalla QED al Modello Standard & sviluppi recenti.

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Elettrodinamica Quantistica

$$\mathcal{L} = -i\overline{\Psi}\not{\mathcal{D}}\Psi - im\overline{\Psi}\Psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \qquad [\not{\mathcal{D}} = \gamma^\mu(\partial_\mu + ieA_\mu)]$$

$\exists$ **due** simmetrie **globali** (la seconda solo per $m = 0$):

- **Fase**: $\begin{cases} \Psi \mapsto e^{+i\Lambda}\Psi \\ \overline{\Psi} \mapsto \overline{\Psi}e^{-i\Lambda} \end{cases}$

Correnti di Noether:

$$J^\mu \propto \overline{\Psi}\gamma^\mu\Psi \qquad \partial_\mu J^\mu = 0$$

La simmetria di fase è **locale**:

$$\boxed{\begin{array}{c} A_\mu \mapsto A_\mu - \partial_\mu\Lambda \\ \mathcal{D}_\mu = \partial_\mu + iA_\mu \end{array}}$$

- **Chirale**: $\begin{cases} \Psi \mapsto e^{+i\Lambda_5\gamma_5}\Psi \\ \overline{\Psi} \mapsto \overline{\Psi}e^{+i\Lambda_5\gamma_5} \end{cases}$

$$J_5^\mu \propto \overline{\Psi}\gamma^\mu\gamma_5\Psi \qquad \partial_\mu J_5^\mu \propto m\overline{\Psi}\gamma_5\Psi$$

Anche se $m = 0$, $\exists$ **anomalia chirale**:

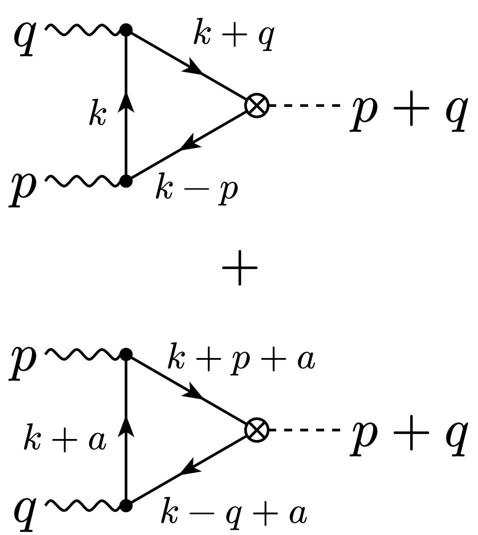
$$\boxed{\partial_\mu J_5^\mu \propto [\hbar]\frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\sigma\rho}}$$

Anomalia a triangolo e Metodo di Fujikawa

$$\boxed{J_5^\mu = \overline{\Psi}(x)\gamma^\mu\gamma_5\mathcal{W}_{x,x+\varepsilon}\Psi(x+\varepsilon)}$$

1. **Calcolo perturbativo**:

Simmetrizziamo e imponiamo $\partial_\mu J_V^\mu = 0$:



2. **Metodo di Fujikawa**:

$$\delta\mathcal{Z} = \delta\int[\mathrm{d}\overline{\Psi}][\mathrm{d}\Psi]e^{i\mathcal{S}} = 0$$

$$\sim \frac{1}{\mathcal{Z}}\int\Big(\boldsymbol{\delta[\mathrm{d}\overline{\Psi}][\mathrm{d}\Psi]}\Big)e^{i\mathcal{S}} - \partial_\mu\langle J_5^\mu\rangle$$

$$[\mathrm{d}\overline{\Psi}][\mathrm{d}\Psi] \simeq \prod_{n,m}\mathrm{d}c_n^\dagger\,\mathrm{d}c_m$$

$$\langle x|\mathrm{tr}[\gamma_5]|x\rangle \mapsto \left\langle x\middle|\mathrm{tr}\left[\gamma_5 e^{-\frac{(i\not{D})^2}{M^2}}\right]\middle|x\right\rangle$$

$$\boxed{\partial_\mu\left\langle\overline{\Psi}\gamma^\mu\gamma_5\Psi\right\rangle = -\frac{e^2}{16\pi^2}\boldsymbol{\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}} := \mathcal{A}\ (\text{anomalia})}$$

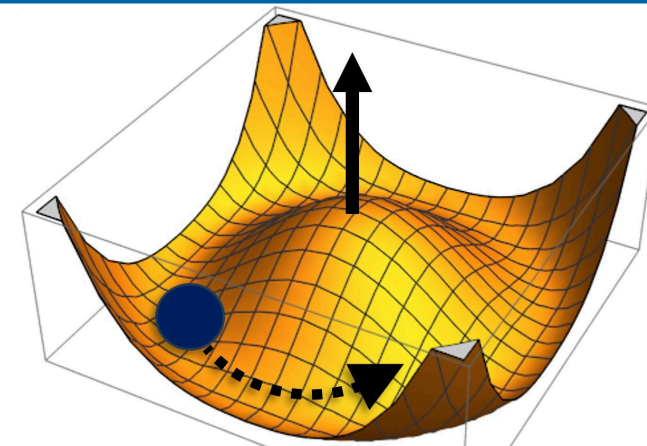
Notare: $\exists J_5^\mu$ conservata perché $\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} = 2\partial_\mu\left[\varepsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma}\right]$

ma non gauge-invariante

Modello Standard e Anomalie

- **Campi di gauge (Y.-M.)**: $(\mathcal{W}^3, \mathcal{B}) \to (\mathcal{Z}_0, \gamma)$
  - Ipercarica $\mathcal{B}$: $\mathrm{U}(1)_Y$
  - Isospin $\mathcal{W}$: $\mathrm{SU}(2)_{\mathrm{w}}$
  - Colore $\mathcal{G}$: $\mathrm{SU}(3)_{\mathrm{s}}$
- **Bosone di Higgs** $\begin{pmatrix}\Phi^+ \\ \Phi^0\end{pmatrix}$ con $\mathrm{Re}(\Phi^0) \to$ Higgs
- **Fermioni**:
  - leptoni $\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L, e^c_L$
  - quark $\begin{pmatrix}u \\ d\end{pmatrix}^\alpha_L, u^{c,\alpha}_L, d^{c,\alpha}_L \qquad \alpha = 1, 2, 3$

$\times$ 3 generazioni identiche (eccetto la massa)



Spontaneous Symmetry Breaking

$$\boldsymbol{Y^3}: \quad \Bigg[2 \cdot \underbrace{\left(-\frac{1}{2}\right)^3}_{(\nu,e)_L} + \underbrace{(1)^3}_{e^c_L} + 6 \cdot \underbrace{\left(\frac{1}{6}\right)^3}_{(u,d)^\alpha_L} + 3 \cdot \underbrace{\left(-\frac{2}{3}\right)^3}_{u^c_L} + 3 \cdot \underbrace{\left(\frac{1}{3}\right)^3}_{d^c_L}\Bigg] = 0$$

$$\boldsymbol{[\mathrm{SU}(2)]^2 Y}: \quad 2 \cdot \Bigg[\underbrace{\left(-\frac{1}{2}\right)}_{(\nu,e)_L} + 3 \cdot \underbrace{\left(\frac{1}{6}\right)}_{(u,d)_L}\Bigg] = 0 \qquad \boldsymbol{[\mathrm{SU}(3)]^2 Y}: \quad 3 \cdot \Bigg[2 \cdot \underbrace{\left(\frac{1}{6}\right)}_{(u,d)_L} + \underbrace{\left(-\frac{2}{3}\right)}_{u^c_L} + \underbrace{\left(\frac{1}{3}\right)}_{d^c_L}\Bigg] = 0$$

$$\boldsymbol{T^2 Y}: \quad \Bigg[2 \cdot \underbrace{\left(-\frac{1}{2}\right)}_{(\nu,e)_L} + \underbrace{(1)}_{e^c_L} + 6 \cdot \underbrace{\left(\frac{1}{6}\right)}_{(u,d)_L} + 3 \cdot \underbrace{\left(-\frac{2}{3}\right)}_{u^c_L} + 3 \cdot \underbrace{\left(\frac{1}{3}\right)}_{d^c_L}\Bigg] = 0$$

Anomalie non abeliane ($D \geq 4$)

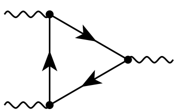
$$D = 4 \quad \Rightarrow \quad \mathcal{D}_\mu \mathcal{J}^\mu_\Lambda \propto \mathrm{tr}[\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma]\, \mathrm{tr}\big[\Lambda\big(\partial_\mu A_\nu \partial_\rho A_\sigma + \dots\big)\big] = \mathcal{A}_\Lambda$$

Wess-Zumino condition: $\quad \delta_\Lambda \mathcal{A}_{\Lambda'} - \delta_{\Lambda'} \mathcal{A}_\Lambda = \mathcal{A}_{[\Lambda,\Lambda']}$

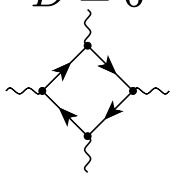
D **dispari**: $\nexists$ anomalie chirali

D **pari**: l'anomalia inizia ad emergere con più vertici:

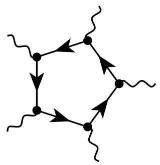
$D = 4$



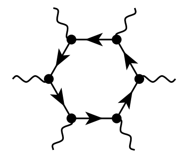
$D = 6$



$D = 8$



$D = 10$



$$p_1{}^{\mu_1} \dots p_{D/2}{}^{\mu_{D/2}} \int \mathrm{d}\Omega\, \mathrm{d}k\, k^{D-1} \frac{\mathrm{tr}[\not{k} \dots + \mathrm{sym}]}{k^D}$$

(Stora-Zumino, 1985): $\quad \mathrm{tr}[\mathcal{F}^n] = \mathrm{d}\omega^{(2n-1)} \qquad \delta_\Lambda \omega^{(2n-1)} = \mathrm{d}\omega^{(2n-2)}_\Lambda \equiv \mathrm{d}\mathcal{A}^{(2n-2)}_\Lambda$

$$D = 10: \quad \mathbf{tr}[\boldsymbol{\mathcal{F}}^6], \quad \mathbf{tr}[\boldsymbol{R}^6], \quad \mathrm{tr}[\mathcal{F}^2]\,\mathrm{tr}[\mathcal{F}^4], \quad \mathrm{tr}[\mathcal{F}^2]^3, \quad \dots \quad (\mathcal{F} \leftrightarrow R)$$

Maxwell (particelle el.):

$$\delta A^{(1)} = -\mathrm{d}\Lambda^{(0)}$$
$$F^{(2)} = \mathrm{d}A^{(1)}$$
$$\mathrm{d} \star F^{(2)} = \star J^{(1)}$$

Higher Forms: (p-brane el.)

$$\delta B^{(p+1)} = -\mathrm{d}\Lambda^{(p)}$$
$$H^{(p+2)} = \mathrm{d}B^{(p+1)}$$
$$\mathrm{d} \star H^{(p+2)} = \star J^{(p+1)}$$

Dualità e.m. ($D - p - 4$-brane mag.):

$$H^{(p+2)} \mapsto \widetilde{H}^{(D-p-2)}$$
$$\mathrm{d} \star \widetilde{H}^{(D-p-2)} = \star \widetilde{J}^{(D-p-3)}$$

$$H^{(3)} = \mathrm{d}B^{(2)} - \omega^{(3)}_A - \omega^{(3)}_R \qquad \delta B^{(2)} = \omega^{(2)}_{\Lambda,A} + \omega^{(2)}_{\Lambda,R} \Rightarrow \mathrm{d}H^{(3)} = -\mathrm{tr}[\mathcal{F} \wedge \mathcal{F}] - \mathrm{tr}[R \wedge R]$$

(Green-Schwarz, 1984) $\mathrm{tr}[\mathcal{F}^2]\,\mathrm{tr}[\mathcal{F}^4], \dots$ sono cancellati da controtermini locali $B\,\mathrm{tr}[\mathcal{F}^4], \dots$

Sviluppi recenti

Motivazioni: **(Gaiotto-Kapustin-Seiberg-Willet, 2015)**

Trattazione comune di simmetrie **continue** e **discrete**

Nuovo ingrediente: **higher-form symmetries** (abeliane):

- connessione $A^{(p+1)}$ e corrente $J^{(p+1)}$
- simmetria $\Lambda^{(p)}$, chiusa se sym globale

$$(D-p)\text{-sottovarietà} \overset{\text{Dualità di Poincaré}}{\Longleftrightarrow} \Lambda^{(p)}\ p\text{-forma}$$

Simmetria espressa in termini di **invarianti topologici** (cfr. Teorema dei Residui)

$$\left\langle e^{i\theta Q(\Sigma_{D-p-1})} O(\Omega_p) \right\rangle = \left\langle U(\theta) O(\Omega_p) \right\rangle = R(\theta)^{\mathbf{Link}(\Sigma_{D-p-1},\Omega_p)} \left\langle O(\Omega_p) \right\rangle$$

Un esempio:

1. Confinamento QCD: parametro d'ordine: $\langle \mathcal{W}[\mathcal{C}] \rangle$ (loop di Wilson)

$\langle \mathcal{W}[\mathcal{C}] \rangle \sim e^{-\text{Area}} \sim 0$	Potenziale confinante	No SSB
$\langle \mathcal{W}[\mathcal{C}] \rangle \sim e^{-\text{Perimetro}} \neq 0$	Potenziale non confinante	**SSB**

$\Rightarrow$ Le higher-form symmetries restringono le regioni accessibili dei diagrammi di fase

Bibliografia

[1] S. L. Adler, *Perturbation Theory Anomalies*. Cambridge, Mass.: MIT Press, 1970.

[2] M. Larosa, «Appunti di Fisica Teorica (A. Sagnotti) — Unpublished». 1998.

[3] B. L. Ioffe, «Axial anomaly: The Modern status», *Int. J. Mod. Phys. A*, vol. 21, pp. 6249–6266, 2006.

[4] M. B. Green, J. H. Schwarz, e E. Witten, *Superstring Theory: 25th Anniversary Edition*. in Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2012.

[5] S. L. Adler, «Axial vector vertex in spinor electrodynamics», *Phys. Rev.*, vol. 177, pp. 2426–2438, 1969.

[6] J. S. Bell e R. Jackiw, «A PCAC puzzle: $\pi^0 \to \gamma\gamma$ in the σ model», *Nuovo Cim. A*, vol. 60, pp. 47–61, 1969.

[7] D. Gaiotto, A. Kapustin, N. Seiberg, e B. Willett, «Generalized Global Symmetries», *JHEP*, vol. 2, p. 172–173, 2015.

[8] L. Bhardwaj *et al.*, «Lectures on generalized symmetries», *Phys. Rept.*, vol. 1051, pp. 1–87, 2024.

[9] P. R. S. Gomes, «An introduction to higher-form symmetries», *SciPost Phys. Lect. Notes*, vol. 74, p. 1–2, 2023.

[10] S. Schafer-Nameki, «ICTP lectures on (non-)invertible generalized symmetries», *Phys. Rept.*, vol. 1063, pp. 1–55, 2024.

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